ATKINSON SCIENCE

THEORY GUIDE

CIBSE Simple Model Web Application

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Atkinson Science welcomes your comments on this Theory Guide. Please send an email to keith.atkinson@atkinsonscience.co.uk.

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1 Introduction

The CIBSE simple model is a steady-state heat loss model of a simple rectangular building. The model does not account for solar gain. Its purpose is to enable the designer to determine the size of convective and radiative heat emitters to achieve a specified operative temperature or a specified air temperature when there is a net heat loss through the building envelope. It is best suited to winter-time calculations in which there is a net heat loss from the building and the solar gain is negligible.

Further information on the theory behind the model can be found in Chapter 5 of Ref. [1].

You can use the model by clicking on this link:
www.atkinsonscience.co.uk/WebApps/Construction/CIBSESimpleModel.aspx

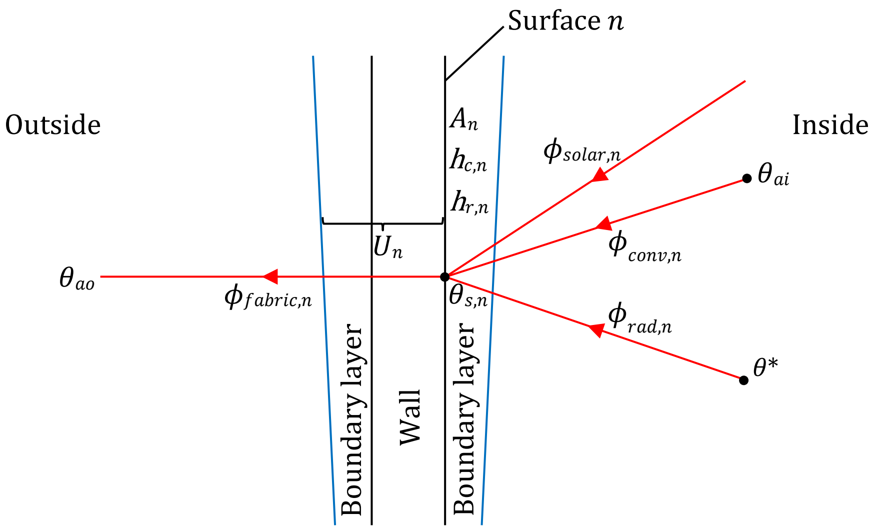
There is also a user guide in PDF format that you can download by clicking on this link:
www.atkinsonscience.co.uk/PDFS/WebApps/CIBSE%20Simple%20Model%20User%20Guide.pdf

2 Theory

2.1 Surface heat balance equation

Figure 1 shows the heat flows at the inner surface of an external wall of a building.

Figure 1 Heat flows at the inner surface of an external wall



In Figure 1, θ_{ao} [C] is the outside air temperature, θ_{ai} [C] is the inside air temperature, $\theta_{s,n}$ [C] is the temperature of wall surface n, and θ^* [C] is a driving radiant temperature reflecting the temperature of the walls and any radiant emitters. The fabric heat loss $\phi_{fabric,n}$ [W] through the surface n is

$$\emptyset_{fabric,n} = \emptyset_{conv,n} + \emptyset_{rad,n} + \emptyset_{solar,n} \quad (2.1)$$

where $\phi_{conv,n}$ [W] is the convection heat transfer at the surface n, $\phi_{rad,n}$ [W] is the radiation absorbed at the surface, and $\phi_{solar,n}$ [W] is the solar radiation absorbed at the surface. The CIBSE simple model is a winter-time model and $\phi_{solar,n}$ can be set to zero (see Ref. [1], page 5-61).

We have

$$\emptyset_{fabric,n} = A_n U_n (\theta_{s,n} - \theta_{ao}) \quad (2.2)$$

$$\emptyset_{conv,n} = A_n h_{c,n} (\theta_{ai} - \theta_{s,n}) \quad (2.3)$$

$$\emptyset_{rad,n} = A_n E_n^* h_{r,n} (\theta^* - \theta_{s,n}) \quad (2.4)$$

Substituting Eqns. (2.3) and (2.4) into Eqn. (2.1) and setting $\phi_{solar,n}$ to zero gives

$$\emptyset_{fabric,n} = A_n U_n(\theta_{s,n} - \theta_{ao})$$

$$= A_n h_{c,n}(\theta_{ai} - \theta_{s,n}) + A_n E_n^* h_{r,n}(\theta^* - \theta_{s,n}) \quad (2.5)$$

We shall assume that all the surface temperatures are known except that of the subject surface. With some simple assumptions we can write

$$E_n^* = \frac{\varepsilon_n}{\left(1 - \varepsilon_n + \frac{5}{6}\varepsilon_n\right)} \quad (2.6)$$

(see Ref [1], page 5-62). By setting ε_n to 1 in the denominator of Eqn. (2.6), we obtain

$$E_n^* = \frac{6}{5}\varepsilon_n \quad (2.7)$$

We shall assume that $h_{c,n}$, $h_{r,n}$ and ε_n are the same on all surfaces. We can sum Eqn. (2.5) over all surfaces to obtain the total fabric loss, Φ_{fabric} [W]:

$$\Phi_{fabric} = \Sigma\emptyset_{fabric,n} = \Sigma A_n U_n \theta_{s,n} - \theta_{ao}\Sigma A_n U_n$$

$$= h_c\theta_{ai}\Sigma A_n - h_c\Sigma A_n\theta_{s,n} + \frac{6}{5}\varepsilon h_r\theta^*\Sigma A_n - \frac{6}{5}\varepsilon h_r\Sigma A_n\theta_{s,n} \quad (2.8)$$

The mean surface temperature θ_m is defined by

$$\theta_m = \frac{\sum A_n\theta_{s,n}}{\sum A_n} \quad (2.9)$$

so

$$\Sigma A_n\theta_{s,n} = \theta_m\Sigma A_n \quad (2.10)$$

Substituting Eqn. (2.10) into Eqn. (2.8) gives

$$\Phi_{fabric} = \Sigma A_n U_n(\theta_m - \theta_{ao})$$

$$= h_c\Sigma A_n(\theta_{ai} - \theta_m) + \frac{6}{5}\varepsilon h_r\Sigma A_n(\theta^* - \theta_m) \quad (2.11)$$

If we define the coefficients

$$H_c = h_c \sum A_n \quad (2.12)$$

and

$$H_r = \frac{6}{5} \varepsilon h_r \sum A_n \quad (2.13)$$

then (2.11) for the fabric heat loss Φ_{fabric} becomes

$$\Phi_{fabric} = \sum (A_n U_n)(\theta_m - \theta_{ao})$$

$$= H_c(\theta_{ai} - \theta_m) + H_r(\theta^* - \theta_m) \quad (2.14)$$

2.2 Ventilation heat loss

A building will also lose heat because of the need for ventilation. In the ventilation process, air is heated from the outside air temperature to the inside air temperature. The air passes through the building and is expelled to the outside. The heat loss due to ventilation Φ_{vent} is

$$\Phi_{vent} = C_{vent}(\theta_{ai} - \theta_{ao}) \quad (2.15)$$

where C_{vent} is the *ventilation conductance*:

$$C_{vent} = \frac{\rho V N_{vent} C_p}{3600}$$

where ρ [kg m^{-3}] is the density of the air, C_p [J kg^{-1} K^{-1}] is the specific heat capacity of the air, V [m^3] is the volume of the building, and N_{vent} [ach] is the number of air changes per hour. We can take ρ to be 1.2 kg m^{-3} and C_p to be 1000 J kg^{-1} K^{-1}, so

$$C_{vent} = \frac{V N_{vent}}{3} \quad (2.16)$$

2.3 Rad-air temperature

The *rad-air temperature* θ_{radair} combines the two driving temperatures θ_{ai} and θ^* and drives both convection and radiation to the surface. It is related to the two driving temperatures by

$$\theta_{radair} = \frac{H_c\theta_{ai}}{H_c + H_r} + \frac{H_r\theta^*}{H_c + H_r}$$

so that the fabric heat loss equation (2.14) becomes

$$\Phi_{fabric} = \sum(A_n U_n)(\theta_m - \theta_{ao})$$

$$= H_c(\theta_{radair} - \theta_m) + H_r(\theta_{radair} - \theta_m)$$

On the inside of a building the rad-air temperature is known as the *environmental temperature* θ_{ei}, so

$$\theta_{ei} = \frac{H_c\theta_{ai}}{H_c + H_r} + \frac{H_r\theta^*}{H_c + H_r} \qquad (2.17)$$

and

$$\Phi_{fabric} = \sum(A_n U_n)(\theta_m - \theta_{ao})$$

$$= H_c(\theta_{ei} - \theta_m) + H_r(\theta_{ei} - \theta_m) \qquad (2.18)$$

2.4 Air temperature and environmental temperature

Collecting together (2.14), (2.15), (2.17) and (2.18), we have

$$\Phi_{fabric} = \sum(A_n U_n)(\theta_m - \theta_{ao})$$

$$= H_c(\theta_{ai} - \theta_m) + H_r(\theta^* - \theta_m) \quad (2.14)$$

$$\Phi_{vent} = C_{vent}(\theta_{ai} - \theta_{ao}) \quad (2.15)$$

$$\theta_{ei} = \frac{H_c \theta_{ai}}{H_c + H_r} + \frac{H_r \theta^*}{H_c + H_r} \quad (2.17)$$

$$\Phi_{fabric} = \sum(A_n U_n)(\theta_m - \theta_{ao})$$

$$= H_c(\theta_{ei} - \theta_m) + H_r(\theta_{ei} - \theta_m) \quad (2.18)$$

We now wish to manipulate these equations to obtain an equation of heat loss in terms of θ_{ai} and an equation of heat loss in terms of θ_{ei}, while eliminating θ^*.

2.4.1 Air temperature

We can simply take the third heat loss term in (2.14), combine it with (2.15), and call the resulting heat loss Φ_a. Both terms represent heat transfer by convection.

$$\Phi_a = C_{vent}(\theta_{ai} - \theta_{ao}) + H_c(\theta_{ai} - \theta_m) \quad (2.19)$$

2.4.2 Environmental temperature

We now need an equation which includes heat transfer by radiation. The radiation heat loss is the last term in (2.14):

$$\Phi_{rad} = H_r(\theta^* - \theta_m)$$

Substituting $H_r\theta^*$ from (2.17) gives

$$\Phi_{rad} = (H_c + H_r)\theta_{ei} - H_c\theta_{ai} - H_r\theta_m$$

Mutiplying both sides by $(H_c + H_r)/H_r$ gives

$$\left(\frac{H_c + H_r}{H_r}\right)\Phi_{rad} = \left(\frac{H_c + H_r}{H_r}\right)(H_c + H_r)\theta_{ei} - \left(\frac{H_c + H_r}{H_r}\right)H_c\theta_{ai} - \left(\frac{H_c + H_r}{H_r}\right)H_r\theta_m$$

$$= \frac{(H_c + H_r)H_c}{H_r}(\theta_{ei} - \theta_{ai}) + (H_c + H_r)(\theta_{ei} - \theta_m)$$

The last term in this equation is the fabric heat loss given by (2.18), so we can write

$$\Phi_e = \left(\frac{H_c + H_r}{H_r}\right)\Phi_{rad}$$

$$= H_a(\theta_{ei} - \theta_{ai}) + \sum(A_n U_n)(\theta_m - \theta_{ao})$$

where

$$H_a = \frac{(H_c + H_r)H_c}{H_r} \qquad (2.20)$$

If we make the approximation $\theta_m = \theta_{ei}$ in the fabric heat loss term then

$$\Phi_e = \sum(A_n U_n)(\theta_{ei} - \theta_{ao}) + H_a(\theta_{ei} - \theta_{ai}) \qquad (2.21)$$

2.5 Standard heat transfer coefficients

We can standardise the heat transfer coefficients h_c and h_r as follows:

$h_c = 3.0$ W m^{-2} K^{-1} (an average figure)

$h_r = 5.7$ W m^{-2} K^{-1} (for temperatures of approximately 20°C)

Also, we can standardise ε with the value 0.9.

Substituting these values into Eqns. (2.12) and (2.13) gives

$$\frac{H_c}{\sum A_n} = h_c = 3.0 \text{ W m}^{-2}\text{ K}^{-1} \qquad (2.22)$$

and

$$\frac{H_r}{\sum A_n} = \frac{6}{5}\varepsilon h_r = \frac{6}{5} \times 0.9 \times 5.7 \cong 6.0 \text{ W m}^{-2}\text{ K}^{-1} \qquad (2.23)$$

Dividing Eqn. (2.20) by $\sum A_n$ gives

$$h_a = \frac{H_a}{\sum A_n} = \frac{(H_c + H_r)H_c}{H_r \sum A_n}$$

and substituting (2.22) and (2.23) gives

$$h_a = \frac{(H_c + H_r)H_c}{H_r \sum A_n} = \frac{\left(h_c \sum A_n + \frac{6}{5}\varepsilon h_r \sum A_n\right) h_c \sum A_n}{\frac{6}{5}\varepsilon h_r \sum A_n \sum A_n}$$

$$= \frac{\left(h_c + \frac{6}{5}\varepsilon h_r\right) h_c}{\frac{6}{5}\varepsilon h_r} = \frac{\left(3.0 + \frac{6}{5} \times 0.9 \times 5.7\right) \times 3.0}{\frac{6}{5} \times 0.9 \times 5.7}$$

$$\cong 4.5 \text{ W m}^{-2}\text{ K}^{-1} \qquad (2.24)$$

3 Development of the CIBSE simple model

We now have an equation of heat loss in terms of θ_{ai}, (2.19), and an equation of heat loss in terms of θ_{ei}, (2.21):

$$\Phi_a = C_{vent}(\theta_{ai} - \theta_{ao}) + h_c \Sigma A_n(\theta_{ai} - \theta_m) \quad (3.1)$$

$$\Phi_e = \Sigma A_n U_n(\theta_{ei} - \theta_{ao}) + h_a \Sigma A_n(\theta_{ei} - \theta_{ai}) \quad (3.2)$$

3.1 Operative temperature

Eqns. (3.1) and (3.2) contain two internal temperatures: the air temperature θ_{ai} and the environmental temperature θ_{ei}. In order to rewrite the equations in terms of a single internal temperature, we introduce the design operative temperature θ_c:

$$\theta_c = ½\theta_{ai} + ½\theta_m$$

The operative temperature is a comfort temperature and represents the temperature felt by occupants of the building. Multiplying the equation for θ_c by 2 and rearranging gives

$$\theta_m = 2\theta_c - \theta_{ai} \quad (3.3)$$

We shall also assume that

$$\theta_{ei} = ⅓\theta_{ai} + ⅔\theta_m \quad (3.4)$$

Eliminating θ_m between (3.3) and (3.4) gives

$$\theta_{ai} = 4\theta_c - 3\theta_{ei} \quad (3.5)$$

Substituting θ_m from Eqn. (3.3) into (3.1) gives

$$\theta_{ai} = \frac{\Phi_a + C_{vent}\theta_{ao} + 2h_c(\sum A_n)\theta_c}{C_{vent} + 2h_c \sum A_n} \quad (3.6)$$

and substituting θ_{ai} from (3.5) into (3.2) gives

$$\theta_{ei} = \frac{\Phi_e + (\sum A_n U_n)\theta_{ao} + 4h_a(\sum A_n)\theta_c}{\sum A_n U_n + 4h_a \sum A_n} \quad (3.7)$$

3.2 Heat source

We shall place a heat source with an output Φ_p [W] in the space. If R ($0 \le R \le 1$) is the radiant fraction of the heat source then the radiant heat output is

$$\Phi_{rad} = \Phi_p R \qquad (3.8)$$

and the convective heat output is

$$\Phi_{conv} = \Phi_p (1 - R) \qquad (3.9)$$

In the CIBSE simple model the heat flow Φ_a required to maintain the air temperature θ_{ai} is

$$\Phi_a = \Phi_{conv} - 0.5\Phi_{rad} = \Phi_p(1 - 1.5R) \qquad (3.10)$$

and the heat flow Φ_e required to maintain the environmental temperature θ_{ei} is

$$\Phi_e = 1.5\Phi_{rad} = 1.5\Phi_p R \qquad (3.11)$$

Note that in (2.21)

$$\Phi_e = \left(\frac{H_c + H_r}{H_r}\right)\Phi_{rad} = \left(\frac{3.0 + 6.0}{6.0}\right)\Phi_{rad} = 1.5\Phi_{rad}$$

Eqns. (3.6) and (3.7) can now be written

$$\theta_{ai} = \frac{\Phi_p(1 - 1.5R) + C_{vent}\theta_{ao} + 2h_c(\sum A_n)\theta_c}{C_{vent} + 2h_c \sum A_n} \qquad (3.12)$$

$$\theta_{ei} = \frac{1.5\Phi_p R + (\sum A_n U_n)\theta_{ao} + 4h_a(\sum A_n)\theta_c}{\sum A_n U_n + 4h_a \sum A_n} \qquad (3.13)$$

Table 1 gives the proportions of radiant and convective heat from various heating systems.

Table 1 Typical proportions of radiant (*R*) and convective heat from heating systems

Heating system	Proportion of emitted radiation	
	Convective	**Radiant (*R*)**
Forced warm air heaters	1.0	0
Natural convectors and convector radiators	0.9	0.1
Multi column radiators	0.8	0.2
Double and treble panel radiators, double column radiators	0.7	0.3
Single column radiators, floor warming systems, storage heaters	0.5	0.5
Vertical and ceiling panel heaters	0.33	0.67
High temperature radiant systems	0.1	0.9

We now have three equations, (3.5), (3.12) and (3.13), containing the outside air temperature θ_{ao}, the inside air temperature θ_{ai}, the environmental temperature θ_{ei}, the operative temperature θ_c, the ventilation conductance C_{vent}, the power of the heat source Φ_p, and the radiant fraction of the heat source R.

The operative temperature θ_c is a comfort temperature, representing the temperature felt by occupants of the space. The CIBSE simple model enables the designer to determine the power of a heat source with a given radiant fraction R needed to achieve either (a) a desired operative temperature θ_c, or (b) a desired air temperature θ_{ai}.

In either case, it is assumed that the outside air temperature θ_{ao}, the ventilation conductance C_{vent} and the radiant fraction R are known. In case (a) the operative temperature θ_c is known and the three equations, (3.5), (3.12) and (3.13), are used to determine the three unknown quantities θ_{ai}, θ_{ei} and Φ_p. In case (b) the air temperature θ_{ai} is known and the three equations are used to determine the three unknown quantities θ_c, θ_{ei} and Φ_p.

We can combine the three equations so as to eliminate two of the unknown quantities, leaving one equation in terms of the power Φ_p. Section 4 deals with case (a) and shows how the equations can be combined to eliminate θ_{ai} and θ_{ei}, leaving an equation for Φ_p in terms of θ_c. Section 5 deals with case (b) and shows how the equations can be combined to eliminate θ_c and θ_{ei}, leaving an equation for Φ_p in terms of θ_{ai}.

4 Achieving the desired operative temperature

In this section we show how to calculate the output needed from a combined convective and radiant heating source with radiant fraction R in order to produce a desired operative temperature. We simply need to manipulate Eqns. (3.5), (3.12) and (3.13).

We can write (3.12) as

$$\theta_{ai} = \frac{\Phi_p(1-1.5R) - C_{vent}\theta_c + C_{vent}\theta_c + C_{vent}\theta_{ao} + 6.0(\sum A_n)\theta_c}{C_{vent} + 6.0\sum A_n}$$

$$= \frac{\Phi_p(1-1.5R) - C_{vent}(\theta_c - \theta_{ao}) + C_{vent}\theta_c + 6.0(\sum A_n)\theta_c}{C_{vent} + 6.0\sum A_n}$$

$$= \frac{\Phi_p(1-1.5R) - C_{vent}(\theta_c - \theta_{ao})}{C_{vent} + 6.0\sum A_n} + \theta_c \qquad (4.1)$$

Similarly, we can write (3.13) as

$$\theta_{ei} = \frac{1.5\Phi_p R - (\sum A_n U_n)\theta_c + (\sum A_n U_n)\theta_c + (\sum A_n U_n)\theta_{ao} + 18.0(\sum A_n)\theta_c}{\sum A_n U_n + 18.0\sum A_n}$$

$$= \frac{1.5\Phi_p R - (\sum A_n U_n)(\theta_c - \theta_{ao}) + (\sum A_n U_n)\theta_c + 18.0(\sum A_n)\theta_c}{(\sum A_n U_n) + 18.0\sum A_n}$$

$$= \frac{1.5\Phi_p R - (\sum A_n U_n)(\theta_c - \theta_{ao})}{\sum A_n U_n + 18.0\sum A_n} + \theta_c \qquad (4.2)$$

Substituting $\theta_{a,i}$ from (4.1) and $\theta_{e,i}$ from (4.2) into Eqn. (3.5) gives

$$\frac{\Phi_p(1-1.5R) - C_{vent}(\theta_c - \theta_{ao})}{C_{vent} + 6.0\sum A_n} + \theta_c = 4\theta_c - 3\left[\frac{1.5\Phi_p R - (\sum A_n U_n)(\theta_c - \theta_{ao})}{\sum A_n U_n + 18.0\sum A_n} + \theta_c\right]$$

which simplifies to

$$\frac{\Phi_p(1-1.5R) - C_{vent}(\theta_c - \theta_{ao})}{C_{vent} + 6.0\sum A_n} = -3\left[\frac{1.5\Phi_p R - (\sum A_n U_n)(\theta_c - \theta_{ao})}{\sum A_n U_n + 18.0\sum A_n}\right] \qquad (4.3)$$

Multiplying (4.3) through by

$$(C_{vent} + 6.0\sum A_n)(\sum A_n U_n + 18.0\sum A_n)$$

gives

$$\begin{aligned}&\left[\Phi_p(1-1.5R) - C_{vent}(\theta_c - \theta_{ao})\right]\left[\sum A_n U_n + 18.0\sum A_n\right] \\ &\qquad = -3.0\left[1.5\Phi_p R - (\sum A_n U_n)(\theta_c - \theta_{ao})\right]\left[C_{vent} + 6.0\sum A_n\right] \qquad (4.4)\end{aligned}$$

By moving the Φ_p terms to the left-hand side of (4.4), we obtain

$$\left[\Phi_p(1-1.5R)\right][\sum A_n U_n + 18.0\sum A_n] + 4.5\Phi_p R[C_{vent} + 6.0\sum A_n]$$
$$= C_{vent}(\theta_c - \theta_{ao})[\sum A_n U_n + 18.0\sum A_n] + 3(\sum A_n U_n)(\theta_c - \theta_{ao})[C_{vent} + 6.0\sum A_n]$$

This equation simplifies to

$$\Phi_p[1.5R(3C_{vent} - \sum A_n U_n) + \sum A_n U_n + 18.0\sum A_n]$$
$$= C_{vent}(\theta_c - \theta_{ao})[\sum A_n U_n + 18.0\sum A_n] + 3(\sum A_n U_n)(\theta_c - \theta_{ao})[C_{vent} + 6.0\sum A_n]$$

so

$$\Phi_p = F_{1cu}(\sum A_n U_n)(\theta_c - \theta_{ao}) + F_{2cu} C_{vent}(\theta_c - \theta_{ao}) \quad (4.5)$$

where

$$F_{1cu} = \frac{3.0(C_{vent} + 6.0\sum A_n)}{1.5R(3C_{vent} - \sum A_n U_n) + \sum A_n U_n + 18.0\sum A_n} \quad (4.6)$$

and

$$F_{2cu} = \frac{\sum A_n U_n + 18.0\sum A_n}{1.5R(3C_{vent} - \sum A_n U_n) + \sum A_n U_n + 18.0\sum A_n} \quad (4.7)$$

Once Φ_p has been calculated, the air temperature θ_{ai} can be calculated from (3.12), the environmental temperature θ_{ei} can be calculated from (3.13), and the mean surface temperature θ_m can be calculated from (3.3).

5 Achieving the desired air temperature

In this section we show how to calculate the output needed from a combined convective and radiant heating source with radiant fraction R in order to produce a desired air temperature. Again, we simply need to manipulate Eqns. (3.5), (3.12) and (3.13).

Multiplying (3.12) through by $C_{vent} + 6.0\sum A_n$ gives

$$C_{vent}\theta_{ai} + 6(\sum A_n)\theta_{ai} = \Phi_p(1 - 1.5R) + C_{vent}\theta_{ao} + 6.0(\sum A_n)\theta_c$$

After moving the Φ_p term to the left-hand side we have

$$\Phi_p(1 - 1.5R) = C_{vent}(\theta_{ai} - \theta_{ao}) + 6(\sum A_n)\theta_{ai} - 6.0(\sum A_n)\theta_c$$

Multiplying the top and bottom of the last two terms in this equation by $\sum A_n U_n + 4.5\sum A_n$ gives

$$\Phi_p(1 - 1.5R) = C_{vent}(\theta_{ai} - \theta_{ao}) + \frac{6(\sum A_n)(\sum A_n U_n + 4.5\sum A_n)\theta_{ai} - 6.0(\sum A_n)(\sum A_n U_n + 4.5\sum A_n)\theta_c}{\sum A_n U_n + 4.5\sum A_n}$$

$$= C_{vent}(\theta_{ai} - \theta_{ao})$$

$$+ \frac{4.5(\sum A_n)}{\sum A_n U_n + 4.5\sum A_n}\left[\frac{4}{3}(\sum A_n U_n)\theta_{ai} + 6.0(\sum A_n)\theta_{ai} - \frac{4}{3}(\sum A_n U_n)\theta_c - 6.0(\sum A_n)\theta_c\right] \quad (5.1)$$

Multiplying (3.13) through by $\sum A_n U_n + 18.0\sum A_n$ gives

$$(\sum A_n U_n + 18.0\sum A_n)\theta_{ei} = 1.5\Phi_p R + (\sum A_n U_n)\theta_{ao} + 18.0(\sum A_n)\theta_c \quad (5.2)$$

Substituting θ_{ei} from (3.5) into (5.2) gives

$$(\sum A_n U_n + 18.0\sum A_n)\left(\frac{4}{3}\theta_c - \frac{1}{3}\theta_{ai}\right) = 1.5\Phi_p R + (\sum A_n U_n)\theta_{ao} + 18.0(\sum A_n)\theta_c$$

or

$$(\sum A_n U_n)\frac{4}{3}\theta_c - (\sum A_n U_n)\frac{1}{3}\theta_{ai} + 18.0(\sum A_n)\frac{4}{3}\theta_c - 18.0(\sum A_n)\frac{1}{3}\theta_{ai}$$
$$= 1.5\Phi_p R + (\sum A_n U_n)\theta_{ao} + 18.0(\sum A_n)\theta_c$$

or

$$(\sum A_n U_n)\frac{4}{3}\theta_c - (\sum A_n U_n)\frac{1}{3}\theta_{ai} + 6.0(\sum A_n)\theta_c - 6.0(\sum A_n)\theta_{ai} = 1.5\Phi_p R + (\sum A_n U_n)\theta_{ao}$$

Rearranging gives

$$-(\sum A_n U_n)\frac{4}{3}\theta_c - 6.0(\sum A_n)\theta_c$$
$$= -(\sum A_n U_n)\frac{1}{3}\theta_{ai} - 6.0(\sum A_n)\theta_{ai} - 1.5\Phi_p R - (\sum A_n U_n)\theta_{ao} \quad (5.3)$$

Substituting (5.3) into (5.1) gives

$$\Phi_p(1-1.5R) = C_{vent}(\theta_{ai}-\theta_{ao}) + \frac{4.5\sum A_n}{\sum A_n U_n + 4.5\sum A_n} \times$$

$$\left[\frac{4}{3}(\sum A_n U_n)\theta_{ai} + 6.0(\sum A_n)\theta_{ai} - (\sum A_n U_n)\frac{1}{3}\theta_{ai} - 6.0(\sum A_n)\theta_{ai} - 1.5\Phi_p R - (\sum A_n U_n)\theta_{ao}\right]$$

or

$$\Phi_p(1-1.5R) = C_{vent}(\theta_{ai}-\theta_{ao}) + \frac{4.5\sum A_n}{\sum A_n U_n + 4.5\sum A_n}\left[(\sum A_n U_n)(\theta_{ai}-\theta_{ao}) - 1.5\Phi_p R\right]$$

Moving the Φ_p terms to the left hand-side gives

$$\Phi_p(1-1.5R) + 1.5\Phi_p R\frac{4.5\sum A_n}{\sum A_n U_n + 4.5\sum A_n}$$
$$= C_{vent}(\theta_{ai}-\theta_{ao}) + \frac{4.5\sum A_n}{\sum A_n U_n + 4.5\sum A_n}(\sum A_n U_n)(\theta_{ai}-\theta_{ao})$$

or

$$\frac{\Phi_p(1-1.5R)(\sum A_n U_n + 4.5\sum A_n) + 1.5\Phi_p R 4.5\sum A_n}{\sum A_n U_n + 4.5\sum A_n}$$
$$= C_{vent}(\theta_{ai}-\theta_{ao}) + \frac{4.5(\sum A_n)}{\sum A_n U_n + 4.5\sum A_n}(\sum A_n U_n)(\theta_{ai}-\theta_{ao})$$

or

$$\frac{\Phi_p(1-1.5R)\sum A_n U_n + 4.5\Phi_p\sum A_n}{\sum A_n U_n + 4.5\sum A_n} = C_{vent}(\theta_{ai}-\theta_{ao}) + \frac{4.5\sum A_n}{\sum A_n U_n + 4.5\sum A_n}(\sum A_n U_n)(\theta_{ai}-\theta_{ao})$$

or

$$\Phi_p = F_{1au}(\sum A_n U_n)(\theta_{ai}-\theta_{ao}) + F_{2au}C_{vent}(\theta_{ai}-\theta_{ao}) \qquad (5.4)$$

where

$$F_{1au} = \frac{4.5\sum A_n}{(1-1.5R)\sum A_n U_n + 4.5\sum A_n} \qquad (5.5)$$

and

$$F_{2au} = \frac{\sum A_n U_n + 4.5\sum A_n}{(1-1.5R)\sum A_n U_n + 4.5\sum A_n} \qquad (5.6)$$

Eqn. (5.3) can be rearranged so that only θ_c is on the left-hand side:

$$\theta_c = \frac{1.5\Phi_p R + \left[\frac{1}{3}\sum A_n U_n + 6.0\sum A_n\right]\theta_{ai} + (\sum A_n U_n)\theta_{ao}}{\frac{4}{3}\sum A_n U_n + 6.0\sum A_n} \qquad (5.7)$$

Once Φ_p has been calculated, the operative temperature θ_c can be calculated from (5.7), the environmental temperature θ_{ei} can be calculated from (3.13), and the mean surface temperature θ_m can be calculated from (3.3).

6 Worked examples

6.1 Small factory

The small factory shown in plan view in Figure 2 is to be heated to an operative temperature of 19°C. The surface areas of the building and the corresponding U values are given in Table 2. The ventilation rate will be 0.5 air changes per hour. The external design temperature is −1°C. Calculate the total heat loss, the inside air temperature and the mean surface temperature when the factory is heated by (a) forced-circulation warm-air heaters, and (b) high-temperature radiant-strip heaters.

Figure 2 Plan view of the factory

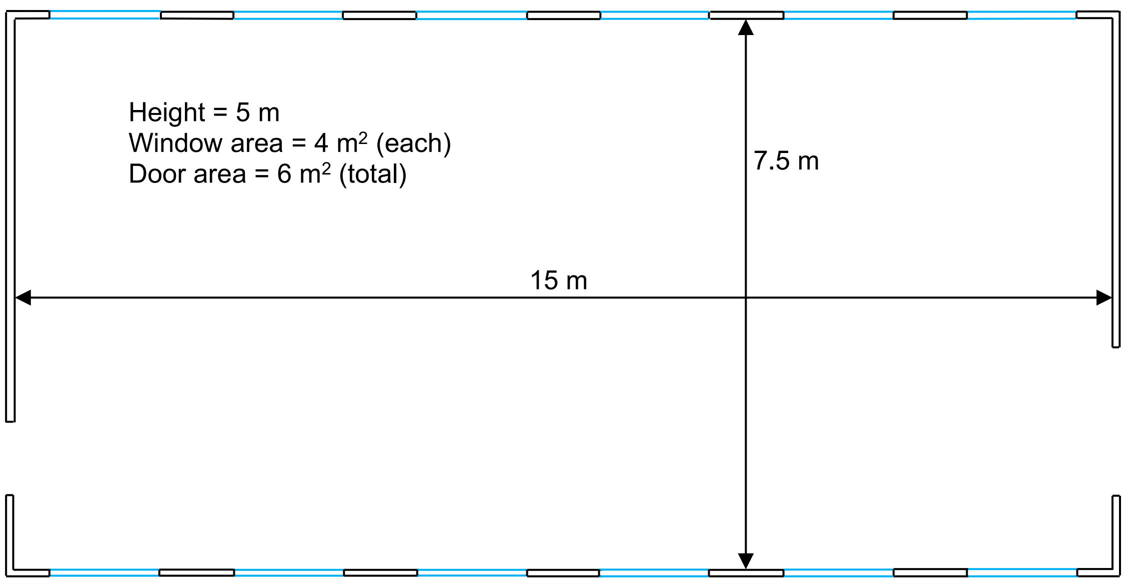


Table 2 Surface areas and *U* values for the factory

Surface	Area A_n [m²]	U_n value [W m⁻² K⁻¹]	A_nU_n [W K⁻¹]
Floor	112.5	0.45	50.6
Roof	112.5	0.3	33.8
External walls	171.0	0.5	85.5
Glazing	48.0	3.3	158.4
Doors	6.0	2.9	17.4
	$\sum A_n = 450.0$		$\sum A_nU_n = 345.7$

(a)

Ventilation conductance

The volume of the building V is 562.5 m³ and the ventilation rate N_{vent} is 0.5 ach. From (2.16) the ventilation conductance C_{vent} is

$$C_{vent} = \frac{N_{vent}V}{3} = \frac{0.5 \times 562.5}{3} = 93.75 \text{ W K}^{-1}$$

Heating power

From Table 1, the radiant fraction R for forced-circulation warm-air heating is zero. Eqn. (4.6) gives the factor F_{1cu}:

$$F_{1cu} = \frac{3.0(C_{vent} + 6.0\sum A_n)}{1.5R(3.0C_{vent} - \sum A_n U_n) + \sum A_n U_n + 18.0\sum A_n}$$

$$= \frac{3.0 \times (93.75 + 6.0 \times 450.0)}{0 + 345.7 + 18.0 \times 450.0} = 0.9923689$$

Eqn. (4.7) gives the factor F_{2cu}:

$$F_{2cu} = \frac{\sum A_n U_n + 18.0\sum A_n}{1.5R(3C_{vent} - \sum A_n U_n) + \sum A_n U_n + 18.0\sum A_n}$$

The first term in the denominator is zero, so F_{2cu} must equal 1. From (4.5)

$$\Phi_p = F_{1cu}(\sum A_n U_n)(\theta_c - \theta_{ao}) + F_{2cu}C_{vent}(\theta_c - \theta_{ao})$$

$$= 0.9923689 \times 345.7 \times \big(19 - (-1)\big) + 1 \times 93.75 \times \big(19 - (-1)\big) = 8{,}736.24 \text{ W}$$

Inside air temperature

The inside air temperature θ_{ai} is given by (3.12):

$$\theta_{ai} = \frac{\Phi_p(1 - 1.5R) + C_{vent}\theta_{ao} + 6.0(\sum A_n)\theta_c}{C_{vent} + 6.0\sum A_n}$$

$$= \frac{8736.24 \times \big(1 - 1.5(0)\big) - 93.75 \times (-1) + 6.0 \times 450.0 \times 19}{93.75 + 6.0 \times 450.0} = 21.5230°\text{C}$$

Mean surface temperature

The mean surface temperature θ_m is given by (3.3):

$$\theta_m = 2\theta_c - \theta_{ai} = 2 \times 19 - 21.5230 = 16.4770°\text{C}$$

(b)

Ventilation conductance

As before, the ventilation conductance C_{vent} is 93.75 W K^{-1}.

Heating power

From Table 1, the radiant fraction R for high-temperature radiant-strip heating is 0.9, so

$$F_{1cu} = \frac{3.0(C_{vent} + 6.0\sum A_n)}{1.5R(3.0C_{vent} - \sum A_n U_n) + \sum A_n U_n + 18.0\sum A_n}$$

$$= \frac{3.0 \times (93.75 + 6.0 \times 450.0)}{1.5 \times 0.9 \times (3.0 \times 93.75 - 345.7) + 345.7 + 18.0 \times 450.0} = 1.0026987$$

and

$$F_{2cu} = \frac{\sum A_n U_n + 18.0\sum A_n}{1.5R(3C_{vent} - \sum A_n U_n) + \sum A_n U_n + 18.0\sum A_n}$$

$$= \frac{345.7 + 18.0 \times 450.0}{1.5 \times 0.9 \times (3.0 \times 93.75 - 345.7) + 345.7 + 18.0 \times 450.0} = 1.0104092$$

From 4.5, the rate of heat supplied is therefore

$$\Phi_p = F_{1cu}(\sum A_n U_n)(\theta_c - \theta_{ao}) + F_{2cu} C_{vent}(\theta_c - \theta_{ao})$$

$$= 1.0026987 \times 345.7 \times \big(19 - (-1)\big) + 1.0104092 \times 93.75 \times \big(19 - (-1)\big) = 8{,}827.18 \text{ W}$$

Inside air temperature

The inside air temperature θ_{ai} is given by (3.12):

$$\theta_{ai} = \frac{\Phi_p(1 - 1.5R) + C_{vent}\theta_{ao} + 6.0(\sum A_n)\theta_c}{C_{vent} + 6.0\sum A_n}$$

$$= \frac{8827.18 \times \big(1 - 1.5(0.9)\big) + 93.75 \times (-1) + 6.0 \times 450 \times 19}{93.75 + 6.0 \times 450.0} = 17.2230°\text{C}$$

Mean surface temperature

The mean surface temperature θ_m is given by Eqn. (3.3):

$$\theta_m = 2\theta_c - \theta_{ai} = 2 \times 19 - 17.2230 = 20.7770°\text{C}$$

6.2 Office

An office is at the corner of an office block on one of the middle floors and has the dimensions shown in Figure 3. The surface areas of the office and the corresponding U values are given in Table 3. The office will have an operative temperature of 21°C and a ventilation rate of 1.2 ach. The external design temperature will be 2°C. The office will be heated by either (a) conventional hot-water radiators, or (b) a warm-air system. In each case calculate the heat load, the inside air temperature and the mean surface temperature.

Figure 3 Dimensions of the office in metres

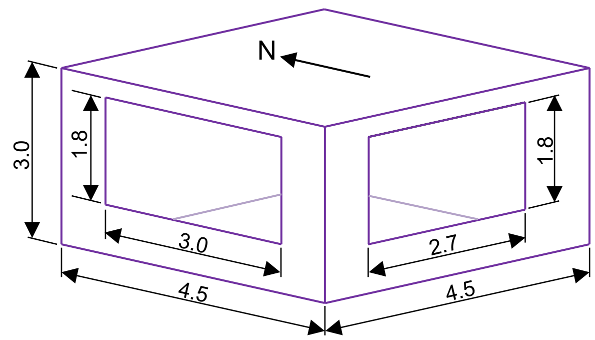


Table 3 Surface areas and *U* values for the office

Surface	Area A_n [m^2]	U_n value [$W\ m^{-2}\ K^{-1}$]	A_nU_n [$W\ K^{-1}$]
W external wall	8.10	0.35	2.835
S external wall	8.64	0.35	3.024
W glazing	5.40	2.2	11.880
S glazing	4.86	2.2	10.692
	$\sum A_n = 27.0$		$\sum A_nU_n = 28.431$

(a)

The ceiling and floor are internal structures so we can assume there is no net heat flow through them. We can make the same assumption about the two internal walls. Consequently, we can ignore these structures when we calculate $\sum A_n$ and $\sum A_n U_n$.

Ventilation conductance

The volume of the office is 4.5 × 4.5 × 3.0 =60.75 m³. The ventilation conductance is given by (2.16):

$$C_{vent} = \frac{VN_{vent}}{3} = \frac{60.75 \times 1.2}{3} = 24.3 \ \text{W K}^{-1}$$

Heating power

From Table 1, the radiant fraction R for multi-column radiators is 0.3, so

$$F_{1cu} = \frac{3.0(C_{vent} + 6.0\sum A_n)}{1.5R(3.0C_{vent} - \sum A_n U_n) + \sum A_n U_n + 18.0\sum A_n}$$

$$= \frac{3.0 \times (24.3 + 6.0 \times 27.0)}{1.5 \times 0.3 \times (3.0 \times 24.3 - 28.431) + 28.431 + 18.0 \times 27.0} = 1.0457635$$

and

$$F_{2cu} = \frac{\sum A_n U_n + 18.0\sum A_n}{1.5R(3C_{vent} - \sum A_n U_n) + \sum A_n U_n + 18.0\sum A_n}$$

$$= \frac{28.431 + 18.0 \times 27.0}{1.5 \times 0.3 \times (3.0 \times 24.3 - 28.431) + 28.431 + 18.0 \times 27.0} = 0.9625571$$

From 4.5, the rate of heat supplied is therefore

$$\Phi_p = F_{1cu}(\sum A_n U_n)(\theta_c - \theta_{ao}) + F_{2cu} C_{vent}(\theta_c - \theta_{ao})$$

$$= 1.0457635 \times 28.431 \times (21 - 2) + 0.9625571 \times 24.3 \times (21 - 2) = 1{,}009.3226 \text{ W}$$

Inside air temperature

The inside air temperature θ_{ai} is given by (3.12):

$$\theta_{ai} = \frac{\Phi_p(1 - 1.5R) + C_{vent}\theta_{ao} + 6.0(\sum A_n)\theta_c}{C_{vent} + 6.0\sum A_n}$$

$$= \frac{1009.3226 \times (1 - 1.5(0.3)) + 24.3 \times 2 + 6.0 \times 27.0 \times 21}{24.3 + 6.0 \times 27.0} = 21.5014°\text{C}$$

Mean surface temperature

The mean surface temperature θ_m is given by (3.3):

$$\theta_m = 2\theta_c - \theta_{ai} = 2 \times 21 - 21.4800 = 20.4985°\text{C}$$

(b)

Ventilation conductance

As before, the ventilation conductance C_{vent} is 24.3 W K^{-1}.

Heating power

From Table 1, the radiant fraction R for a forced-convection warm-air system is 0, so

$$F_{1cu} = \frac{3.0(C_{vent} + 6.0\sum A_n)}{1.5R(3.0C_{vent} - \sum A_n U_n) + \sum A_n U_n + 18.0\sum A_n}$$

$$= \frac{3.0 \times (24.3 + 6.0 \times 27.0)}{1.5 \times 0 \times (3.0 \times 24.3 - 28.431) + 28.431 + 18.0 \times 27.0} = 1.0864431$$

and

$$F_{2cu} = \frac{\sum A_n U_n + 18.0\sum A_n}{1.5R(3C_{vent} - \sum A_n U_n) + \sum A_n U_n + 18.0\sum A_n}$$

The first term in the denominator is zero, so F_{2cu} must equal 1.

From 4.5, the rate of heat supplied is therefore

$$\Phi_p = F_{1cu}(\sum A_n U_n)(\theta_c - \theta_{ao}) + F_{2cu} C_{vent}(\theta_c - \theta_{ao})$$

$$= 1.0864431 \times 28.431 \times (21 - 2) + 1 \times 24.3 \times (21 - 2) = 1{,}048.5846 \text{ W}$$

Inside air temperature

The inside air temperature θ_{ai} is given by (3.12):

$$\theta_{ai} = \frac{\Phi_p(1 - 1.5R) + C_{vent}\theta_{ao} + 6.0(\sum A_n)\theta_c}{C_{vent} + 6.0\sum A_n}$$

$$= \frac{1048.5846 \times \left(1 - 1.5(0)\right) + 24.3 \times 2 + 6.0 \times 27.0 \times 21}{24.3 + 6.0 \times 27.0} = 24.1502°\text{C}$$

Mean surface temperature

The mean surface temperature θ_m is given by (3.3):

$$\theta_m = 2\theta_c - \theta_{ai} = 2 \times 21 - 24.1502 = 17.8497°\text{C}$$

7 References

1. *CIBSE Guide A: Environmental Design*, CIBSE, 2006.